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Fourth Order Nonlinear Evolution Equations For Counter Propagating Capillary Gravity Wave Packets In The Presence Of Wind Flowing Over Water

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Abstract: **Asymptotically exact and nonlocal fourth order nonlinear evolution equations are derived for two counter propagating surface capillary gravity wave packets in deep water in the presence of wind flowing bover water. On the basis of these evolution equations stability analysis is made for a uniform standing surface capillary gravity wave trains for longitudinal perturbation. Instability condition is obtained and graphs are plotted for maximum growth rate of instability and for wave number at marginal stability against wave steepness for some different values of dimensionless wind velocity. Significant deviations are noticed from the results obtained from third order nonlinear evolution equations.**

Keywords: **Nonlinear evolution equation, capillary gravity, waves, stability analysis.**

I. INTRODUCTION

There has been considerable interest in the stability of finite amplitude surface gravity wave in deep water.Much of this interest has been focused on the instability of a uniform wave train to modulational perturbations.For small but finite amplitude, one successful approach to studying the stability of finite amplitude surface gravity waves in deep water is through the application of the lowest order nonlinear evolution equation,which is the nonlinear Schrodinger equation.This analysis is suitable for small wave steepness and for long-wavelength perturbations.But for wave steepness greater than 0.15 predictions from the nonlinear Schrodinger equation do not agree with the result of Longuet-Higgins[12,13]. Dysthe[3] has shown that astability analysis made from a fourth-order nonlinear evolution equation that is one order higher than the nonlinear Schrodinger equation gives results consistent with the exact results of Longuet-Higgins[12,13] and with the experimental results of Benjamin and Feir[1] for wave steepness up to 0.25.The fourth-order effects give a surprising improvement compared to ordinary nonlinear Schrodinger effects in many respects,and some of these points have been elaborated by Janssen [9].The dominant new effect that comes in the fourth order is the influence of wave-induced mean flow and this produces a significant deviation in the stability character. From these it can be concluded that a fourth-order evolution equation is a good starting point for studying nonlinear effects in surface waves. Fourth order nonlinear evolution equation for deep water surface waves in different contexts and stability analysis made from them were derived by Dhar and Das[4,5,6]. Debsarma and Das [7],Hara and Mei[10,11],Bhattacharyya and Das[2].

II. BASIC EQUATIONS

The common horizontal interface between water and air in the undisturbed state as z=0 plane.In the undisturbed state air flows over water with a velocity u in a direction that is taken as the x- axis. We take $z=\eta(x,y,t)$ as the equation of the common interface is at any time t in the perturbed state. We introduce the dimensional quantities $\tilde{\phi},\tilde{\phi}',\tilde{\eta},(\tilde{x},\tilde{y},\tilde{t}),\tilde{t},\tilde{v},\tilde{\gamma}$ and

$\tilde{s}$ which are respectively, the perturbed velocity potential in water, perturbed velocity potential in air, surface elevation of the water-air interface, space coordinates, time, air flow velocity, the ratio of the densities of air to water and surface tension.

These dimensionless quantities are related to the corresponding dimensional quantities by the following relations

$$\left.\begin{aligned} &\tilde{\phi} = \sqrt{k_0^3/g}\ \phi, \quad \tilde{\phi}' = \sqrt{k_0^3/g}\ \phi', \quad (\tilde{x},\tilde{y},\tilde{z}) = (k_0x, k_0y, k_0z), \\ &\tilde{\eta} = k_0\eta, \tilde{t} = \omega t, \tilde{v} = \sqrt{k_0/g}\ v, \quad \tilde{\gamma} = \frac{\rho'}{\rho}, \ \tilde{s} = Tk^2{}_0/g, \end{aligned}\right\} \tag{1}$$

where k_0 is some characteristic wave number, g is the acceleration due to gravity, ρ and ρ' are the densities of water and air respectively and T is the dimension surface tension.

In the future, all the quantities will be written in their dimensionless form with their over (~) dropped.

The perturbed velocity potentials ϕ and ϕ' satisfy the following Laplace equations

$$\nabla^2\phi = 0 \qquad \text{in} \qquad -\infty < z < \eta \tag{2}$$

$$\nabla^2\phi' = 0 \qquad \text{in} \qquad \eta < z < \infty \tag{3}$$

The kinematic boundary conditions to be satisfied at the interface are the following

$$\frac{\partial\phi}{\partial z} - \frac{\partial\eta}{\partial t} = \frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y} \qquad \text{when} \qquad z = \eta \tag{4}$$

$$\frac{\partial\phi'}{\partial z} - \frac{\partial\eta}{\partial t} - v\frac{\partial\eta}{\partial x} = \frac{\partial\phi'}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial\phi'}{\partial y}\frac{\partial\eta}{\partial y}, \qquad \text{when} \quad z = \eta \tag{5}$$

The condition of continuity of pressure at the interface gives

$$\begin{aligned} \frac{\partial\phi}{\partial t} - \gamma\frac{\partial\phi'}{\partial t} + (1-\gamma)\eta - \gamma v\frac{\partial\phi'}{\partial x} &= \frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right\} \\ &+ \frac{\gamma}{2}\left\{\left(\frac{\partial\phi'}{\partial x}\right)^2 + \left(\frac{\partial\phi'}{\partial y}\right)^2 + \left(\frac{\partial\phi'}{\partial z}\right)^2\right\} + s\left\{1 + \left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial y}\right)^2\right\}^{\frac{-3}{2}} \\ &\times\left\{\left(\frac{\partial\eta}{\partial x}\right)^2\frac{\partial^2\eta}{\partial y^2} + \left(\frac{\partial\eta}{\partial y}\right)^2\frac{\partial^2\eta}{\partial x^2} - 2\frac{\partial\eta}{\partial x}\frac{\partial\eta}{\partial y}\frac{\partial^2\eta}{\partial x\partial y} + \frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right\}, \end{aligned}$$

$$\text{when} \quad z = \eta \tag{6}$$

Also ϕ and ϕ' should satisfy the following conditions at infinity

$$\frac{\partial\phi}{\partial z} \to 0 \qquad \text{when} \qquad z \to -\infty \tag{7}$$

$$\frac{\partial\phi'}{\partial z} \to 0 \qquad \text{when} \qquad z \to \infty \tag{8}$$

We look for solutions of the above equations (2) and (8)in the following form

$$G = G_{00} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[G_{mn} \exp i(m\psi_1 + n\psi_2) + G^*_{mn} \exp - i(m\psi_1 + n\psi_2)] \tag{9}$$

where $\psi_1 = k_1 x - \omega\, t,\ \ \psi_1 = k_1 x - \omega\, t$ and G stands for $\phi, \phi'\ \eta$. In the above summation on the right hand side of equation (9), $(m,n) \neq (0,0)$. The Fourier coefficients $\phi_{00},\ \phi'_{00},\ \phi_{mn},\ \phi'_{mn},\ \phi^*_{mn}\ \phi'^*_{mn},$ are functions of $z,\ \ x_1 = \varepsilon x, t_1 = \varepsilon\, t$ and $\eta_{00},\ \eta_{mn},\ \eta^*_{mn},$ are functions of $x_1, y_1\ t_1$. ε is a small ordering parameter measuring the weakness of wave steepness which is the product of wave amplitude and wave number and * denotes complex conjugate.

The linear dispersion relation for gravity waves

$$(1+\gamma)\omega^2 - 2\gamma\,\omega k\, v + \gamma k^2 v^2 - (1-\gamma) - s = 0 \tag{10}$$

which gives the following two values of $\omega_\pm$

$$\omega_\pm = \left[\gamma v \pm \sqrt{1-\gamma^2 - \gamma v^2 + s\left(1+\gamma\right)}\right] / \left(1+\gamma\right) \tag{11}$$

that corresponds to two modes and we designate this two modes as positive and negative modes. The positive mode moves in the positive direction of the x -axis with a frequency $\left[\gamma v + \sqrt{1-\gamma^2 - \gamma v^2 + s\left(1+\gamma\right)}\right] / \left(1+\gamma\right)$, while the negative mode moves in the negative direction of the x -axis with a frequency $\left[\sqrt{1-\gamma^2 - \gamma v^2 + s\left(1+\gamma\right)} - \gamma v\right] / \left(1+\gamma\right)$. If v is replace by $-v$ the frequency of the positive mode becomes equal to the frequency of the negative mode. So the results for the negative mode can be obtained from those for the positive mode by replacing v by $-v$. Therefore we have made a nonlinear analysis for the positive mode, and then we have obtained the results for the negative mode by replacing v by $-v$.

For linear stability v should satisfy the following condition

$$|v| < \sqrt{\left[1-\gamma^2 + s\left(1+\gamma\right)\right] / \gamma} \qquad 12)$$

So our present analysis will remain valid as long as the dimensionless flow velocity of the air becomes less than the critical velocity $\sqrt{\left[1-\gamma^2 + s\left(1+\gamma\right)\right] / \gamma}$. For air flowing over water γ = 0.00129 and this critical value becomes 28.87, s=0.075.

III. DERIVATION OF EVOLUTION EQUATIONS

Substituting expansions (9) in equations (2),(3),(7),(8) and then equating the coefficients of $\exp i(m\psi_1 + n\psi_2)$ for (m,n)=[(1,0),(0,1),(2,0),(0,2),(1,1),(-1,1)]

we obtain the following equations:

$$\left(\frac{\partial^2}{\partial z^2} - \Delta^2_{mn}\right)\phi_{mn} = 0 \tag{13}$$

$$\left(\frac{\partial^2}{\partial z^2} - \Delta^2_{mn}\right)\phi'_{mn} = 0 \tag{14}$$

$$\frac{\partial \phi_{mn}}{\partial z} \to 0 \tag{15}$$

$$\frac{\partial \phi'_{mn}}{\partial z} \to 0 \tag{16}$$

where Δ_{mn} is the operator given by

$$\Delta^2_{mn} = \left\{(m+n) - i\varepsilon \frac{\partial}{\partial x_1}\right\}^2 - \varepsilon^2 \frac{\partial^2}{\partial {y_1}^2} \tag{17}$$

The solutions of equations(13) and(14) satisfying boundary conditions (15)and(16) respectively are given by

$$\phi_{mn} = \exp(\Delta_{mn} z) A_{mn} \tag{18}$$

$$\phi'_{mn} = \exp(\Delta_{mn} z) A'_{mn} \tag{19}$$

in which A_{mn}, A'_{mn} are functions of x_1, y_1 and t_1. For the sake of convenience we take the Fourier transformation of equations (2),(3),(7) and (8) for (m,n)=(0,0). The solutions of these transformed

equations becomes

$$\bar{\phi}_{00} = \exp(|\bar{k}|z)\bar{A}_{00} \tag{20}$$

$$\bar{\phi}'_{00} = \exp(|\bar{k}|z)\bar{A}'_{00} \tag{21}$$

where $\bar{\phi}_{00}$ and $\bar{\phi}'_{00}$ are Fourier transforms of ϕ_{00} and ϕ'_{00} respectively, defined by

$$(\bar{\phi}_{00}, \bar{\phi}'_{00}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\bar{\phi}_{00}, \bar{\phi}'_{00}) \exp i(\bar{k}_x x_1 + \bar{k}_y y_1 - \bar{\varpi} t_1) dx_1 dy_1 dt_1 \tag{22}$$

where $\bar{k}^2 = (\bar{k}_x{}^2 + \bar{k}_y{}^2)$, $\bar{A}_{00}$ and $\bar{A}'_{00}$ are functions of $\bar{k}_x$ and $\bar{k}_y$ and $\bar{\varpi}$.

Again substituting expansions (9) in the Taylor expanded forms of equations (4)-(6) about z=0 and then equating the coefficients of $\exp i(m\psi_1 + n\psi_2)$ for (m,n)=[(1,0),(0,1),(2,0),(0,2),(1,1),(-1,1),(0,0)] on both sides , we get the following equations

$$\left(\frac{\partial \phi_{mn}}{\partial z}\right)_{z=0} + i\left\{(m-n)\omega + i\varepsilon \frac{\partial}{\partial t_1}\right\}\eta_{mn} = p_{mn} \tag{23}$$

$$\left(\frac{\partial \phi_{mn}}{\partial z}\right)_{z=0} + i\left\{(m-n)\omega + i\varepsilon \frac{\partial}{\partial t_1}\right\}\eta_{mn} - iv\left\{(m+n)\omega - i\varepsilon \frac{\partial}{\partial x_1}\right\}\eta_{mn} = q_{mn} \tag{24}$$

$$-i\left\{(m-n)\omega + i\varepsilon \frac{\partial}{\partial t_1}\right\}(\phi_{mn})_{z=0} + i\gamma\left\{(m-n)\omega + i\varepsilon \frac{\partial}{\partial t_1}\right\}(\phi'_{mn})_{z=0} + (1-\gamma)\eta_{mn}$$

$$+ s\Delta_{mn}\eta_{mn} - i\gamma v\left\{(m+n)\omega - i\varepsilon \frac{\partial}{\partial x_1}\right\}(\phi'_{mn})_{z=0} = r_{mn} \tag{25}$$

where $(\ )_{z=0}$ implies the value of the quantity inside brackets at z=0 and p_{mn}, q_{mn}, r_{mn} are contributions from nonlinear terms. Now for the above seven values of (m,n), we obtain seven sets of equations,in which we substitute the solutions for ϕ_{mn}, ϕ'_{mn} given by (18)-(21). We now considering the following perturbation expansions for the solutions of above three sets of equations

$$P_{mn} = \sum_{i=1}^{\infty} \varepsilon^i P_{mn}^i \text{ for (m,n)=[(1,0),(0,1)] ; } P_{mn} = \sum_{i=1}^{\infty} \varepsilon^i P_{mn}^i \text{ for (m,n)=[(2,0),(0,2),(1,1),(-1,1),(0,0)]} \tag{26}$$

where P_{mn} stands for A_{mn} , A'_{mn} and η_{mn} .

Substituting expansions (26) in the above three sets of equations and then equating coefficients of various powers of ε on both sides,we obtain a sequence of equations. From the first order (i.e lowest order) and second order equations corresponding to (23) and (24) of the first set of equations we obtain solutionsfor A_{mn} , A'_{mn} and η_{mn} ; (m,n)=)=[(1,0),(0,1), (2,0),(0,2),(1,1),(-1,1),(0,0)].We now take the following transformations, following Pierce and Knobloch[14]of all perturbed quantities in slow spacecoordinates and time given by $\xi_+ = x_1 - c_g t_1,\quad \xi_- = x_1 + c_g t_1,\quad \varsigma = y_1,\quad \tau_2 = \varepsilon^2 t_1$ where $c_g = (\frac{dw}{dk})_{k=1}$ is the group velocity. As we are going to derive evolution equation correct up to $O(\varepsilon^4)$ which is one order higher than the evolution equation in the lowest order, we have introduced one more slow time variable τ_2 following Weissman [15] and we get the fourth order evolution equation for η_{10} :

$$\frac{\partial \eta_{10}^{(1)}}{\partial \tau_1} + \delta_1 \frac{\partial^2 \eta_{10}^{(1)}}{\partial \xi_+^2} + \delta_2 \frac{\partial^2 \eta_{10}^{(1)}}{\partial \varsigma^2} + \varepsilon \left[i \frac{\partial \eta_{10}^{(1)}}{\partial \tau_2} + i \frac{\partial \left\langle \eta_{10}^{(2)} \right\rangle_-}{\partial \tau_1} + \delta_1 \frac{\partial^2 \left\langle \eta_{10}^{(2)} \right\rangle_-}{\partial \xi_+^2} + \delta_2 \frac{\partial^2 \left\langle \eta_{10}^{(2)} \right\rangle_-}{\partial \varsigma^2} + i\delta_4 \frac{\partial^3 \eta_{10}^{(1)}}{\partial \xi_+^3} + i\delta_5 \frac{\partial^3 \eta_{10}^{(1)}}{\partial \xi_+ \partial \varsigma^2} \right]$$
$$= \gamma_1 |\eta_{10}^{(1)}|^2 \eta_{10}^{(1)} + \gamma_2 |\eta_{10}^{(1)2}| \eta_{10}^{(1)} + \varepsilon \left[\gamma_1 \eta_{10}^{(1)2} \left\langle \eta_{10}^{(2)*} \right\rangle_- + 2\gamma_1 |\eta_{10}^{(1)}|^2 \left\langle \eta_{10}^{(2)} \right\rangle_- + \gamma_2 \eta_{10}^{(1)} \left\langle \eta_{01}^{(1)} \eta_{10}^{(2)*} \right\rangle_- + \gamma_2 \eta_{10}^{(1)} \left\langle \eta_{01}^{(1)*} \eta_{10}^{(2)} \right\rangle_- \right]$$
$$+\varepsilon \left[\gamma_2 \left\langle \eta_{10}^{(2)} |\eta_{01}^{(1)}|^2 \right\rangle_- + i\gamma_3 |\eta_{10}^{(1)}|^2 \frac{\partial \eta_{10}^{(1)}}{\partial \xi_+} + i\gamma_4 \eta_{10}^{(1)2} \frac{\partial \eta_{10}^{(1)*}}{\partial \xi_+} + i\gamma_7 \left\langle |\eta_{01}^{(1)}|^2 \right\rangle_- \frac{\partial \eta_{10}^{(1)}}{\partial \xi_+} + \gamma_8 \eta_{10}^{(1)} + 2\eta_{10}^{(1)} H \frac{\partial}{\partial \xi_+} \left(|\eta_{10}^{(1)}|^2 \right) \right]$$

where (27)

$$\gamma_8 = i\gamma_5 \left\langle \eta_{01}^{(1)} \frac{\partial \eta_{01}^{(1)*}}{\partial \xi_-} \right\rangle_- + i\gamma_6 \left\langle \eta_{01}^{(1)*} \frac{\partial \eta_{01}^{(1)}}{\partial \xi_-} \right\rangle_- - 2 \left\langle H \frac{\partial}{\partial \xi_+} \left(|\eta_{10}^{(1)}|^2 \right) \right\rangle_-$$

Again we get the fourth order evolution equation for η_{01} :

$$\frac{\partial \eta_{01}^{(1)}}{\partial \tau_1} + \delta_1 \frac{\partial^2 \eta_{01}^{(2)}}{\partial \xi_-^2} + \delta_2 \frac{\partial^2 \eta_{01}^{(1)}}{\partial \varsigma^2} + \varepsilon \left[i \frac{\partial \eta_{01}^{(1)}}{\partial \tau_2} + i \frac{\partial \left\langle \eta_{01}^{(2)} \right\rangle_-}{\partial \tau_1} + \delta_1 \frac{\partial^2 \left\langle \eta_{01}^{(2)} \right\rangle_-}{\partial \xi_+^2} + \delta_2 \frac{\partial^2 \left\langle \eta_{01}^{(2)} \right\rangle_-}{\partial \varsigma^2} + i\delta_4 \frac{\partial^3 \eta_{01}^{(2)}}{\partial \xi_+^3} + i\delta_5 \frac{\partial^3 \eta_{01}^{(1)}}{\partial \xi_+ \partial \varsigma^2} \right]$$
$$= \gamma_1 |\eta_{01}^{(1)}|^2 \eta_{01}^{(1)} + \gamma_2 |\eta_{01}^{(1)2}| \eta_{01}^{(1)} + \varepsilon \left[\gamma_1 \eta_{10}^{(1)2} \left\langle \eta_{10}^{(2)*} \right\rangle_- + 2\gamma_1 |\eta_{10}^{(1)}|^2 \left\langle \eta_{10}^{(2)} \right\rangle_- + \gamma_2 \eta_{10}^{(1)} \left\langle \eta_{01}^{(1)} \eta_{01}^{(2)*} \right\rangle_- + \gamma_2 \eta_{01}^{(1)} \left\langle \eta_{01}^{(1)*} \eta_{01}^{(2)} \right\rangle_- \right]$$
$$+\varepsilon \left[\gamma_2 \left\langle \eta_{01}^{(2)} |\eta_{10}^{(1)}|^2 \right\rangle_+ + i\gamma_3 |\eta_{10}^{(1)}|^2 \frac{\partial \eta_{10}^{(1)}}{\partial \xi_-} + i\gamma_4 \eta_{01}^{(1)2} \frac{\partial \eta_{01}^{(1)*}}{\partial \xi_-} + i\gamma_7 \left\langle |\eta_{01}^{(1)}|^2 \right\rangle_+ \frac{\partial \eta_{01}^{(1)}}{\partial \xi_-} + \gamma_9 \eta_{01}^{(1)} + 2\eta_{01}^{(1)} H \frac{\partial}{\partial \xi_-} \left(|\eta_{10}^{(1)}|^2 \right) \right]$$

where (28)

$$\gamma_9 = i\gamma_5 \left\langle \eta_{10}^{(1)} \frac{\partial \eta_{10}^{(1)*}}{\partial \xi_+} \right\rangle_+ + i\gamma_6 \left\langle \eta_{10}^{(1)*} \frac{\partial \eta_{10}^{(1)}}{\partial \xi_+} \right\rangle_+ - 2\left\langle H \frac{\partial}{\partial \xi_+} \left(|\eta_{10}^{(1)}|^2 \right) \right\rangle_-$$

If we put $\varepsilon = 0, v = 0, \gamma = 0$ in equation (27) and (28) then we get nonlocal mean field evolution equations in the third order for infinite depth water. These reduce equations becomes the same as equations(1b) of Pierce and Knobloch[14] when we proceed to the limit as $h \to \infty$.

IV. STABILITY OF FINITE AMPLITUDE WAVE TRAINS

The uniform wave train solutions of equations (27) and (28) are given by

$$\eta_{10} = \eta_{10}^{(0)} = \alpha_0 \exp(i\Delta\omega\tau), \quad \eta_{01} = \eta_{01}^{(0)} = \alpha_0 \exp(i\Delta\omega\tau), \tag{29}$$

where α_0 is real constant and the nonlinear frequency shift $\Delta\omega$ is given by

$$\Delta\omega = -(\gamma_1 + \gamma_2)\alpha_0^2 \tag{30}$$

Finally we obtain the following nonlinear dispersion relation

$$\Omega = \left\{\delta_1\lambda^2 \left(\delta_1\lambda^2 + 2\gamma_1\alpha_0^2\right)\right\}^{\frac{1}{2}} - \varepsilon\left\{\delta_4\lambda^2 + \lambda\alpha_0^2\left(\gamma_3 + \gamma_7\right)\right\} - 2\varepsilon\delta_1\lambda^2 \,|\,\lambda\,|\,\alpha_0^2 / \{\delta_1\lambda^2(\delta_1\lambda^2 + 22\gamma_1\alpha_0^2)\}^{\frac{1}{2}} \tag{31}$$

Solving for $\lambda^2 = -\gamma_1\alpha_0^2 / \delta_1$, the maximum growth rate of instability I_m is given by

$$I_m = |\,\gamma_1\,|\,\alpha_0^2 - \frac{2\varepsilon\gamma_1\alpha_0^3}{\sqrt{|\,\gamma_1\delta_1\,|}} \tag{33}$$

and at marginal stability $$\lambda = \frac{\sqrt{2}\gamma_1\alpha_0}{\sqrt{|\,\gamma_1\delta_1\,|}} \tag{34}$$

In Figures 1 and 2 the maximum growth rate I_m of instability which can be obtained from equation (33) has been plotted against wave steepness α_0 for some different values of dimensionless wind velocity v and for s=0.075.From the above graphs it is found that in the fourth order analysis for waves with sufficiently small wave numbers the maximum growth rate of instability I_m first increases with the increase of wave steepness α_0 and then it decreases with the increase of α_0 and finally vanishes at some critical value of wave steepness α_0 beyond which there is no instability, while in the third order analysis the maximum growth rate of instability I_m increases steadily with the increase of wave steepness α_0 .The growth rate is found to be appreciably much higher for dimensionless wind velocity approaching its critical value. Again in Figure 3 the wave number λ at marginal stability which can be obtained from equation (34) has been plotted against wave steepness α_0 for some different values of dimensionless wind velocity v. From these figures it is observed that the instability regions are shortened with the increase of the absolute value of wind velocity.

V. DISCUSSION AND CONCLUSION

The third order nonlinear evolution equations have been derived by Pierce and Knobloch[14] for two counterpropagating capillary gravity wave packets on the surface of water of finite depth.The resulting equations are asymptotically exact and nonlocal and generalize the equations derived by Djordjevic andRedekopp [8] for counterpropagating waves.Our paper is an extension of the evolution equations derived by Pierce and knobloch[14] to one order higher for an infinite depth water and in the presence of wind flowing over it.The reason for starting from a fourth order nonlinear evolution equation is motivated by the fact, as shown by Dysthe[3]that a fourth order nonlinear evolution equation is a good starting point for making stability analysis of a uniform wave train in deep water.The evolution equations derived by us have been used toinvestigate the stability of a uniform standing wave train under longitudinal perturbations.Instability condition is obtained and graphs are plotted showing maximum growth rate of instability against wave steepness for some different values of dimensionless wind velocity v. From the graphs it is found that in the fourth order analysis for waves with sufficiently small wave numbers the maximum growth rate of instability first increases with the increase of wave steepness and then it decreases with the increase of wave steepness and finally vanishes at some critical value of wave steepness beyond which there is no instability,while in the third order analysis the maximum growth rate of instability increases steadily with the increase of wave steepness The growth rate of instability is found to be appreciably much higher for dimensionless wind velocity approaching its critical value. Our results show significant deviations from the results obtained from third order nonlinear evolution equations.Graphs are also plotted for the wave number at marginal stability against wave steepness for some different values of dimensionless wind velocity v. From the graphs it is observed that the instability regions are shortened with the increase of the absolute value of wind velocity.

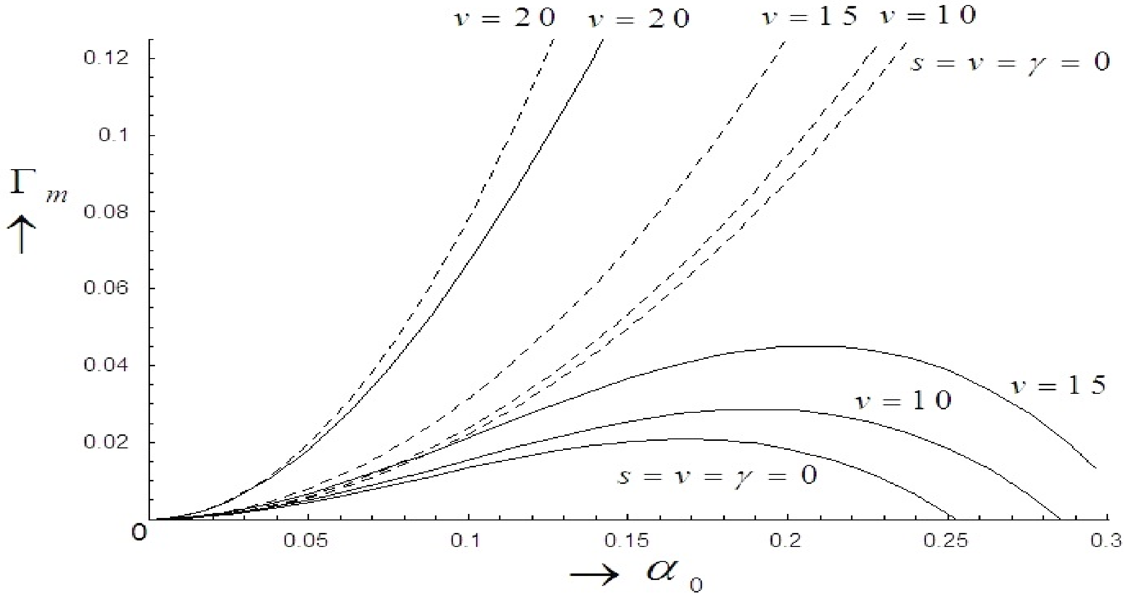

Figure 1: Maximum growth rate of instability I_m against wave steepness α_0 for some different values of dimensionless wind velocity v .Here $\gamma = 0.00129$ and s=0.075 for all the graphs except for two with $v = \gamma = s = 0$ written on the graph. ___________ fourth order results -------------------third order results.

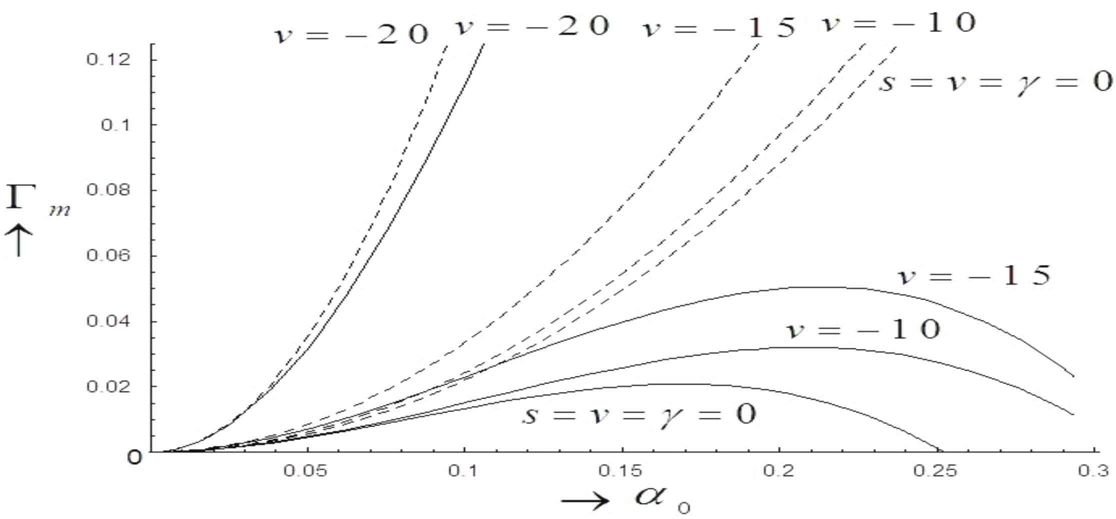

Figure 2: Maximum growth rate of instability I_m against wave steepness α_0 for some different values of dimensionless wind velocity v. Here $\gamma = 0.00129$ and s=0.075 for all the graphs.. ________ fourth order results --------------third order results.

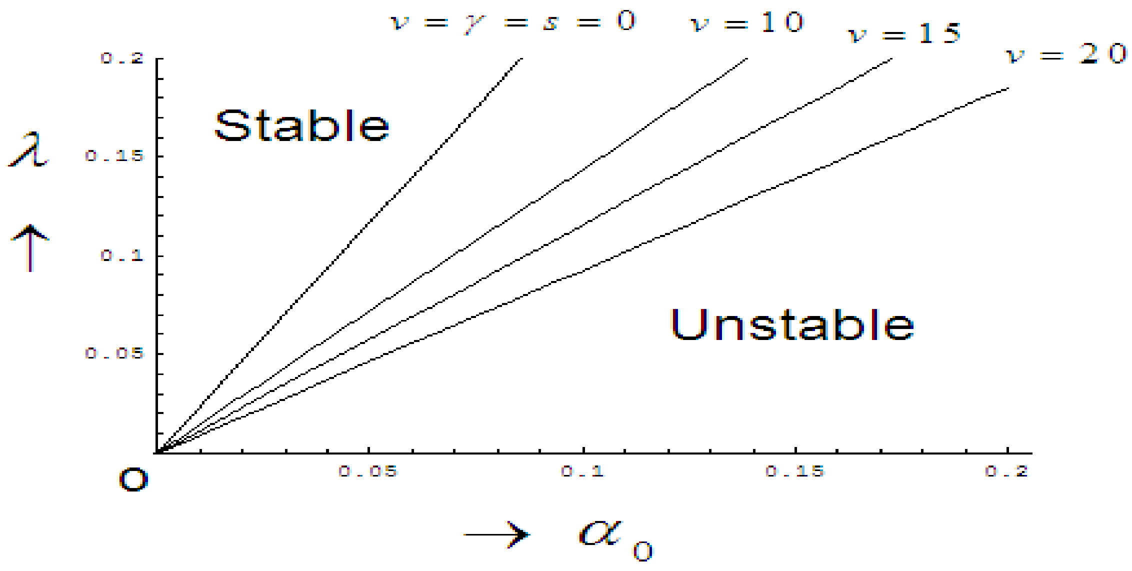

Figure 3

APPENDIX: COEFFICIENTS OF THE EVOLUTION EQUATIONS (27) AND (28).}

$$\delta_0 = \left\{2\gamma v\omega - 2\gamma v^2 + (1-\gamma) + 3s\right\} / \left\{(1+\gamma)\omega^2 - \gamma v\omega\right\}, \delta_1 = \left\{2\gamma v c_g - \gamma v^2 + (1+\gamma)c_g^2 + 3s\right\} / \left\{2(1+\gamma)\omega^2 - 2\gamma v\omega\right\},$$

$$\delta_2 = \left\{(1-\gamma)c_g^2 - 2\gamma v c_g + 3s\right\} / \left\{4(1+\gamma)\omega^2 - 4\gamma v\omega\right\}, \quad \delta_3 = \left\{2\omega c_g - (1+\gamma)c_g^2 + \gamma(\omega^2 - v\omega) + 3s\right\} / \left\{(1+\gamma)\omega^2 - \gamma v\omega\right\},$$

$$\delta_4 = \left\{\omega^2(4c_g^2 - 1) - 9s^2 + (1 - 5s^2 - s^3)(1-\gamma) + \omega v(1+s^2) - 2\gamma v\omega\right\} / 4\omega^2[2\omega^2(1+\gamma) - 2\omega v]^2$$

$$\delta_5 = \left\{\omega^2(4c_g^2 - 1) + (9+s)(1-\gamma) + \gamma v\omega c_g\right\} / 6\omega^2[2\omega^2(1+\gamma) - 2\omega v]^2$$

$$\gamma_1 = [(2\omega^4 + 6\omega^2 - 9s) + \gamma\{\frac{21}{2}(\omega^2 + v^2) + 2(2+p_1)(\omega - v)(\omega - v - 2\omega^2)$$
$$-(1+2p_1)\omega + 15\omega v\} + \gamma v(\omega - v)(6p_1 + 9)] / [12\omega^2 - 8\omega^4 - \gamma(\omega - v)^2],$$

$$\gamma_2 = [31\omega^4 - 23\omega^2 + s^2(1-\gamma) - 8s + 8\gamma(\omega - v)^2 - \gamma v(8 - \omega p_2)] / [8\omega^4 - 6\omega^2 - 2\gamma(\omega - v)^2],$$

$$\gamma_3 = [12s^3(4\omega^2 c_g^2 - 5s) - (108s^3 + 27s^2 - 3s + 24)(1-\gamma) + 2\gamma v(\omega + v + p_1\omega)] / [\omega^2\{\omega^2(1+\gamma) + 2\gamma v^2 - 3(1-\gamma)\}^2],$$

$$\gamma_4 = [2s(6 + 4s + s^2)(1-\gamma) - (4\omega^2 c_g^2 - \omega^2 + 8) - \gamma v(\omega + v)] / [2\omega^2(1+\gamma) + 2\gamma v^2 - 3(1-\gamma)\}^2],$$

$$\gamma_5 = [(4\omega^2 c_g^2 - 5s + 1) - 13s^2(1-\gamma) + 3\gamma v(\omega - v - p_1 v\omega)] / [2\omega^2(1+\gamma) + 2\gamma v^2 - 3(1-\gamma)\}^2],$$

$$\gamma_6 = [30s(4\omega^2 c_g^2 - 1) + (1+\gamma)(2s^3 + 98s^2 + 103s + 10) + \gamma(p_1\omega + v) + \gamma v w(\omega - v)] / [2\omega^2\{4\omega^2(1+\gamma) + 2\gamma v^2 - 3(1-\gamma)\}^2],$$

$$\gamma_7 = [94(4\omega^2 c_g^2 - \omega^2 - 5s) + (448s^4 + 111s^3 + 3s^2 + 259 + 24)(1+\gamma) + \gamma v(p_1\omega - v)(\omega - v)]$$
$$/ [4\omega^2\{4\omega^2(1+\gamma) + 2\gamma v^2 - 3(1-\gamma)\}^2],$$

where

$$c_g = [2\gamma v\omega - 2\gamma v^2 + (1-\gamma) + 3s] / [2(1+\gamma)\omega - 2\gamma v],$$

$$p_1 = [\omega^2 - 3\gamma(\omega - v)^2] / [2\omega^2(\gamma - 1) - 2\gamma\omega(v + \omega) + 3(1-\gamma)\}],$$

$$p_2 = [2\omega^2(1+\gamma) - 4\gamma v^2] / [2\omega^2(\gamma + 1) + 2\gamma\omega(\omega - v) - 3(1-\gamma)\}],$$

Nomenclature:

, $\left.\begin{array}{l}\delta_i^{(1)}(i=1,2,3)\\ \gamma_i^{(1)}(i=1,2,3,4)\\ \beta_i^{(1)}(i=1,2,3,4,5)\end{array}\right\}$ - coefficients given in the Appendix,

ε - Slowness parameter, α - wave steepness, ζ - elevation of the air water interface, ω - frequency, , γ - ratio of densities of air to water, $\Delta\omega$ - frequency shift, Ω - perturbed frequency at marginal stability.

g – acceleration due to gravity H – Hilbert's transform operator, λ - wave number, s – dimensionless surface tension, t – time, v - air flow velocity, I_m -growth rate of instability .

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